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ADIABATIC PHYSICO-MATHEMATICAL MODEL OF THE VUILLEUMIER THERMAL-ACTED HEAT PUMP

BY

VLAD MARIO HOMUTESCU, EMIL JUGUREANU and ADRIAN HOMUTESCU

Abstract: The paper presents an adiabatic model of the Vuilleumier machine. The model assumes the hypothesis that inside the low and high temperature chambers and inside the intermediate temperature chambers adiabatic processes take place only. Inside the heat exchangers only isothermal processes take place. The differential equations of the processes inside the heat pump are deduced.

Key words: Vuilleumier machine, adiabatic model, thermal-acted heat pump

1. Introduction

A thermal-acted heat pump (a Vuilleumier machine) [1], [2], [3], [4], [5] is a machine inside which a constant amount of gas evolves inside an almost constant total volume. The gas lies inside several heat exchangers and four variable volume chambers placed (most often) inside two cylinders, each cylinder being fit with its own displacer piston. There are three levels of temperature inside the machine. The machine receives heat at the lower temperature of the cycle by expanding the gas inside the low temperature chamber. Pressure variation inside the machine is acquired by heating the agent inside a high temperature chamber and by cooling the agent inside two intermediate temperature chambers.

2. Vuilleumier Machine Construction and Functioning

Accordingly to the schematic diagram in fig. 1 [1], [2], [3], [4], a Vuilleumier machine is comprised of a cold cylinder 1 and a hot cylinder 15 inside which the cold displacer 3 and the hot displacer 13 work. The cold cylinder and displacer share a diameter inferior to the one shared by the hot cylinder and displacer. A drive comprised of crankshaft 17 and rods 16 and 18 provide movement for the displacers. The cold displacer splits the space inside its cylinder in two chambers: a low temperature one 4 and an intermediate temperature one 2. Inside the hot cylinder the hot displacer delimitates a high temperature chamber 12 and an intermediate temperature chamber 14.

Each cylinder is fit with its own heat exchanger set. The cold cylinder has a low temperature heater 5, a low temperature regenerator 6 and an intermediate temperature cooler 7. The hot cylinder is fit with an intermediate temperature cooler 9, a hot temperature regenerator 10 and a high temperature heater 11. The intermediate temperature cooling chambers are connected through pipe 8.

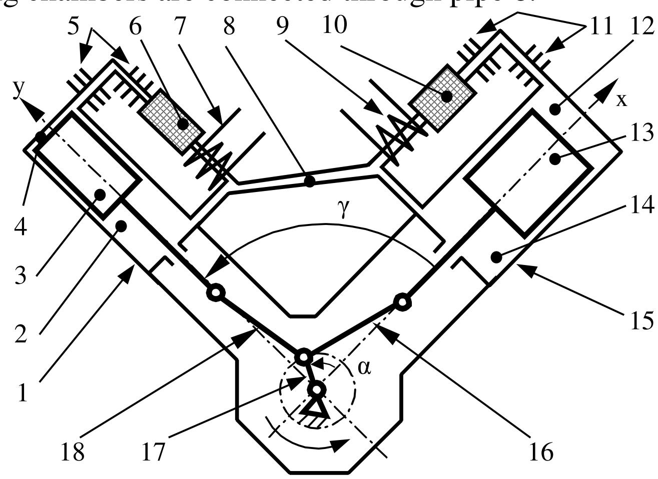

Fig. 1. Vuilleumier machine.

3. The Hypotheses of the Model

The adiabatic model is based on the following hypotheses:

● the working agent is the ideal gas,

● the gas amount inside the machine is constant,

● at thermodynamic level all cycle functional processes are time independent,

● the metallic parts of the machine (other than the heaters and the coolers) do not exchange heat either among them or with the exterior,

● the processes inside heat regenerators are ideal ones (regeneration efficiencies are 100%); the agent temperature inside the regenerator is deemed constant, being taken as arithmetic or logarithmic mean,

● inside the low temperature chamber 4 (on fig. 1), inside the high temperature chamber 12 and inside the intermediate temperature chambers 2 and 14 adiabatic processes take place; so, the temperature inside these chambers vary cyclically,

● the agent temperature inside the coolers is equal to the one of the outer cooling agent, the one of the cylinder walls next to their respective chambers and the one of the stems and of the displacer bottoms; the agent temperature inside the heaters is equal to the one of the outer heating agent, the one of the cylinder walls next to their respective chambers and the one of the displacer heads,

● inside the coolers and heaters only isothermal processes take place,

● the instantaneous pressure is identical in all the spaces occupied by the agent,

● the movement of the displacers is the real movement, given by the crankshaft.

The hypotheses implying the temperatures inside the Vuilleumier machine show that inside four of the machine chambers take place adiabatic processes and inside all other chambers isothermal processes take place only, thus confirming the described physico-mathematical model the denomination of adiabatic model. To outline the adiabatic character of the physico-mathematical model analyzed here, on fig. 2 the machine chambers are separated and placed in row. This presentation required the halving of each displacer. Each variable volume chamber is assigned half a displacer. The mechanical linkage between the displacer halves was symbolically drawn through bars exterior to the cylinder.

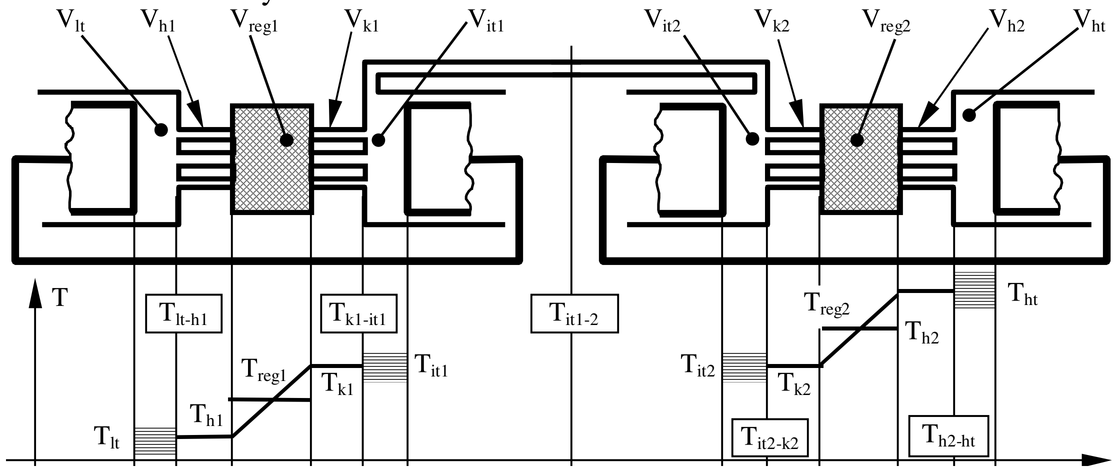

Fig. 2. Adiabatic model of the Vuilleumier machine.

The following subscripts for dimensions inside machine chambers (volume V, temperature T, mass m, enthalpy I) were used: h = heater; reg = regenerator; k = cooler; 1 = cold displacer; 2 - hot displacer; ht = high temperature; lt = low temperature; it = intermediate temperature; T = total. The composed subscripts lt-h1, k1-it1, it1-2, it2-k2 and h2-ht refer to the dimensions describing the separating sections between the adiabatic chambers and their neighboring chambers.

4. The Adiabatic Physico-mathematical Model

The model uses the differential equation of the conservation of the working agent total mass, the equation of state applied to the heat exchangers and the differential law of conservation of energy written for the adiabatic chambers [6].

The differential equation of the conservation of the working agent total mass is

$$\begin{aligned} d(m_T) = d(m_{lt}) + d(m_{h1}) + d(m_{reg1}) + d(m_{k1}) + d(m_{it1}) \\ + d(m_{it2}) + d(m_{k2}) + d(m_{reg2}) + d(m_{h2}) + d(m_{ht}) = 0 . \end{aligned} \tag{1}$$

The differential expressions of the agent masses inside the heat exchangers are obtained from the equation of state, in which V = const. and T = const.:

$$\frac{dp}{p} = \frac{dm}{m} . \tag{2}$$

The mass m is taken from the equation of state and the differential expression of the mass inside a generic heat exchanger becomes:

$$dm_j = \frac{V_j}{R\,T_j} dp , \tag{3}$$

where the subscript j is replaced by h1, reg1, k1, k2, reg2 and h2.

Accordingly to the adopted hypotheses, inside the low temperature chamber, inside the intermediate temperature chambers and inside the high temperature chamber the gas exchanges work with the surrounding environment (through piston movement) and enthalpy with the neighboring chambers (through the agent's entering the chamber from the neighboring heat exchangers or leaving it toward the heat exchangers). The internal energy of the gas inside the adiabatic chamber changes, as a consequence of mass and temperature variations. Inside these chambers the heat exchanged is zero, conforming to the adiabatic hypothesis. The energy balance takes the expression:

(4) $$dL + dU + dI = 0 .$$

For the low temperature chamber the terms in (4) are explicated by the relations:

(5) $$dL_{lt} = p\,dV_{lt} ,$$

(6) $$dU_{lt} = d(c_v\, m_{lt}\, T_{lt}) = \frac{c_v}{R}(V_{lt}\, dp + p\, dV_{lt}) ,$$

(7) $$dI_{lt-h1} = d(c_p\, m_{lt-h1}\, T_{lt-h1}) = -c_p\, T_{lt-h1}\, dm_{lt} .$$

Equation (7) takes into account that $dm_{lt-h1} = -dm_{lt}$, because the mass of working agent that passes through the section lt-h1 is equal to the variation of the mass of the gas inside the chamber, taken with opposite sign. The positive sense of the agent flow inside the machine is considered to be from the low temperature chamber toward the high temperature chamber. The term $c_p\, m_{lt}\, dT_{lt-h1}$ was neglected, assuming the hypothesis that it is small in comparison with the other term.

Introducing (5), (6) and (7) in (4) and explaining the mass differential, the following relation is obtained:

(8) $$dm_{lt} = \frac{1}{R\, T_{lt-h1}}\left[p\, dV_{lt} + \frac{V_{lt}}{k} dp \right].$$

Similarly, for the other three adiabatic chambers the next relations are obtained:

(9) $$dm_{ht} = \frac{1}{R\, T_{h2-ht}}\left[p\, dV_{ht} + \frac{V_{ht}}{k} dp \right],$$

(10) $$dm_{it1} = \frac{dp}{R}\left[\frac{V_{it1}}{k\, T_{it1-2}} + \left(\frac{T_{k1-it1}}{T_{it1-2}} - 1 \right)\left(\frac{V_{lt}}{k\, T_{lt-h1}} + \frac{V_{h1}}{T_{h1}} + \frac{V_{reg1}}{T_{reg1}} + \frac{V_{k1}}{T_{k1}} \right) \right] + \\ + \frac{p}{R\, T_{it1-2}} dV_{it1} + \left(\frac{T_{k1-it1}}{T_{it1-2}} - 1 \right) \frac{p}{R\, T_{lt-h1}} dV_{lt} ,$$

(11) $$dm_{it2} = \frac{dp}{R}\left[\frac{V_{it2}}{k\, T_{it1-2}} + \left(\frac{T_{it2-k2}}{T_{it1-2}} - 1 \right)\left(\frac{V_{ht}}{k\, T_{h2-ht}} + \frac{V_{h2}}{T_{h2}} + \frac{V_{reg2}}{T_{reg2}} + \frac{V_{k2}}{T_{k2}} \right) \right] + \\ + \frac{p}{R\, T_{it1-2}} dV_{it2} + \left(\frac{T_{it2-k2}}{T_{it1-2}} - 1 \right) \frac{p}{R\, T_{h2-ht}} dV_{ht} .$$

Introducing the mass differentials for the ten chambers of the machine, given by (3), (8), (9), (10) and (11) in (1), the differential expression of the pressure is obtained:

(12) $$dp = A / B ;$$

(13) $$A = -k\, p\left(dV_{it1} + dV_{it2} + \frac{T_{k1-it1}}{T_{lt-h1}} dV_{lt} + \frac{T_{it2-k2}}{T_{h2-ht}} dV_{ht} \right) ;$$

(14) $$B = V_{it1} + V_{it2} + \frac{T_{k1-it1}}{T_{lt-h1}} V_{lt} + \frac{T_{it2-k2}}{T_{h2-ht}} V_{ht} + k\, T_{k1-it1}\left(\frac{V_{h1}}{T_{h1}} + \frac{V_{reg1}}{T_{reg1}} + \frac{V_{k1}}{T_{k1}} \right) + \\ + k\, T_{it2-k2}\left(\frac{V_{h2}}{T_{h2}} + \frac{V_{reg2}}{T_{reg2}} + \frac{V_{k2}}{T_{k2}} \right),$$

where the terms A and B are given by (13) and (14).

For the adiabatic chambers the differential expression of the temperature is taken from the equation of state:

$$\frac{dp}{p}+\frac{dV}{V}=\frac{dm}{m}+\frac{dT}{T}. \tag{15}$$

Particularizing for the four chambers, the following relations are obtained

$$dT_{lt}=T_{lt}\left(\frac{dp}{p}+\frac{dV_{lt}}{V_{lt}}-\frac{dm_{lt}}{m_{lt}}\right), \tag{16}$$

$$dT_{it1}=T_{it1}\left(\frac{dp}{p}+\frac{dV_{it1}}{V_{it1}}-\frac{dm_{it1}}{m_{it1}}\right), \tag{17}$$

$$dT_{it2}=T_{it2}\left(\frac{dp}{p}+\frac{dV_{it2}}{V_{it2}}-\frac{dm_{it2}}{m_{it2}}\right), \tag{18}$$

$$dT_{ht}=T_{ht}\left(\frac{dp}{p}+\frac{dV_{ht}}{V_{ht}}-\frac{dm_{ht}}{m_{ht}}\right). \tag{19}$$

Equations (8), (9), (10), (11), (12), (16), (17), (18) and (19) form the system of differential equations of the adiabatic physico-mathematical model of the Vuilleumier machine. The unknown functions are the pressure p, the masses m_{lt} , m_{it1}, m_{it2} and m_{ht} inside the adiabatic chambers and the temperatures T_{lt}, T_{it1}, T_{it2} and T_{ht} in the same chambers. The system is non-linear, because there are several terms in the differential equations that have an order higher than one. The system has variable coefficients and the conditional temperatures $T_{lt\text{-}h1}$, $T_{k1\text{-}it1}$, $T_{it1\text{-}2}$, $T_{it2\text{-}k2}$ and $T_{h2\text{-}ht}$ of the agent that passes through the surfaces lt-h1, k1-it1, it1-2, it2-k2 and h2-ht depend on the sense of the gas flow. The conditional temperatures take the expressions:

$$\begin{cases}T_{lt-h1}=T_{lt} \\ T_{lt-h1}=T_{h1}\end{cases} \text{if} \quad \begin{matrix} m_{lt-h1}\ge 0 \\ m_{lt-h1}<0\end{matrix} \quad \text{or} \quad \begin{matrix} dm_{lt}\le 0 \\ dm_{lt}>0\end{matrix}$$

$$\begin{cases}T_{k1-it1}=T_{k1} \\ T_{k1-it1}=T_{it1}\end{cases} \text{if} \quad \begin{matrix} m_{k1-it1}\ge 0 \\ m_{k1-it1}<0\end{matrix} \quad \text{or} \quad \begin{matrix} d(m_{lt}+m_{h1}+m_{reg1}+m_{k1})\le 0 \\ d(m_{lt}+m_{h1}+m_{reg1}+m_{k1})>0\end{matrix}$$

$$\begin{cases}T_{it1-2}=T_{it1} \\ T_{it1-2}=T_{it2}\end{cases} \text{if} \quad \begin{matrix} m_{it1-2}\ge 0 \\ m_{it1-2}<0\end{matrix} \quad \text{or} \quad \begin{matrix} d(m_{lt}+m_{h1}+m_{reg1}+m_{k1}+m_{it1})\le 0 \\ d(m_{lt}+m_{h1}+m_{reg1}+m_{k1}+m_{it1})>0\end{matrix} \tag{20}$$

$$\begin{cases}T_{it2-k2}=T_{it2} \\ T_{it2-k2}=T_{k2}\end{cases} \text{if} \quad \begin{matrix} m_{it2-k2}\ge 0 \\ m_{it2-k2}<0\end{matrix} \quad \text{or} \quad \begin{matrix} d(m_{k2}+m_{reg2}+m_{h2}+m_{ht})\ge 0 \\ d(m_{k2}+m_{reg2}+m_{h2}+m_{ht})<0\end{matrix}$$

$$\begin{cases}T_{h2-ht}=T_{h2} \\ T_{h2-ht}=T_{ht}\end{cases} \text{if} \quad \begin{matrix} m_{h2-ht}\ge 0 \\ m_{h2-ht}<0\end{matrix} \quad \text{or} \quad \begin{matrix} dm_{ht}\ge 0 \\ dm_{ht}<0\end{matrix}$$

The system can be solved only by numerical integration (e.g. a Runge-Kutta method). The solution can be obtained after several iterations.

As a consequence of the displacer stem's presence, the machine exchanges with the environment an amount of work per cycle. The heat amounts Q_{h1} , Q_{k1} , Q_{k2} and Q_{h2} exchanged in the machine are calculated from the energy balance.

The coefficient of performance of the heat pump is

$$\varepsilon_{hp}=\left|Q_{k1}+Q_{k2}\right|/Q_{h2}. \tag{21}$$

5. Conclusions

The physico-mathematical adiabatic model proposed for the numerical simulation of the Vuilleumier machine functioning allows for providing information on the possible performance the machine is capable of. Inside a real machine the heat exchanges do not take place isothermally, the heat regeneration is not ideal and the agent flow through the heat exchangers occurs with friction, all these facts lowering the performance beneath the adiabatic one.

The model stresses the heat amounts exchanged inside the heaters and coolers. Accordingly to the adopted hypotheses, the heaters and the coolers (there are adjacent to the adiabatic chambers) cyclically exchange nonzero heats.

The heat amounts exchanged with the heat sources using the adiabatic model are larger than the corresponding ones calculated with the isothermal model. The coefficient of performance (COP) is smaller than the isothermal one.

The adiabatic model allows for an analysis of the influence that some constructive and functional factors have (more than the isothermal model can support) as well as for comparing different machines.

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Technical University of Iassy
Department of Thermodynamics and Thermal Machines

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MODEL FIZICO-MATEMATIC ADIABATIC AL POMPEI DE CĂLDURĂ CU ACŢIONARE TERMICĂ VUILLEUMIER

(Rezumat)

Lucrarea prezintă deducerea sistemului de ecuaţii diferenţiale ce descriu funcţionarea pompei de căldură cu acţionare termică Vuilleumier pe baza ipotezelor modelului adiabatic. S-a admis că în camerele cu volum variabil nu se schimbă căldură cu exteriorul şi că în toate camerele cu volum constant au loc procese la temperatură constantă.